Hydrodynamic Dispersive and Advective Processes in Watershed Responses
Francisco Olivera, P.E., M.ASCE, 1 and Srikanth Koka 2

Abstract: A model for estimating the watershed response is presented and used for assessing how the watershed size and spatial variability of the hydrodynamic parameters (i.e., wave celerity and dispersion coefficient) affect the comparative importance of advective processes with respect to hydrodynamic dispersive processes. A parameter $\Omega$ was defined to quantify this comparative importance. This parameter represents the fraction of the watershed response variance that is explained by advection. A series of simulations were performed for basins of different sizes and different spatial distributions of their hydrologic parameters. It was found that, for spatially uniform hydrodynamic parameters, the effect of hydrodynamic dispersion decreases compared to that of advection as the watershed size increases, and vice versa; and that, for nonuniform hydrodynamic parameters, the spatial distribution of the parameter values over the watershed, in conjunction with the watershed size, determines which process—advection or hydrodynamic dispersion—prevails.


CE Database subject headings: Hydrologic models; Surface runoff; Watershed management; Geomorphology; Wave dispersion; Geographic information systems.

Introduction

The hydrologic response of a watershed to a precipitation event is the result of a series of physical processes that take place in the drainage area. In this paper, a model for estimating the watershed response is presented and used for assessing the comparative importance of some of these physical processes with respect to others. In particular, this paper addresses how the watershed size and spatial variability affect the comparative importance of advective processes with respect to hydrodynamic dispersive processes. Advective processes refer to the translation of water with its cross-sectional average flow velocity; while hydrodynamic dispersive processes account for the fact that, because of turbulence, shear stresses and/or floodplain storage, at any given cross section, some water particles flow faster than others.

In the last thirty years, models of the watershed response based on the river network geomorphology have been discussed in the literature (Kirkby 1976; Rodriguez-Iturbe and Valdes 1979; Gupta et al. 1980; Wang et al. 1981; Troutman and Karlinger 1985; Mesa and Mifflin 1986; van der Tak and Bras 1990; Rinaldo et al. 1991; Naden 1992; Beven and Wood 1993; Snell and Sivapalan 1994; Troch et al. 1994; Rinaldo et al. 1995; Robinson et al. 1995; Wooding 1995; Saco and Kumar 2002a,b; Botter and Rinaldo 2003; D’Odorico and Rigon 2003). These methods, as opposed to the unit hydrograph model (Sherman 1932) widely used in small watersheds, account for the fact that some water particles travel along long flowpaths to reach the watershed outlet (i.e., watershed outskirts), while others travel along short flow paths (i.e., outlet vicinity). However, even though they are very suitable for supporting spatially variable hydrodynamic parameters, not all of them do so and, if they do, only in a limited fashion.

Results from Saco and Kumar’s (2002b) research, for example, show that, as the watershed size increases, the effect of hydrodynamic dispersion on the watershed response increases at a faster rate than that of the river network geomorphology. These results, however, are “contrary to the postulate of Rinaldo et al. (1991),” who argues the opposite and whose work has been of reference to many researchers in the last decade. This discrepancy illustrates the need and timeliness of further research on the physical processes that affect watershed responses.

In this paper, a watershed response model that supports the use of nonuniform hydrodynamic parameters and excess precipitation is presented. A series of simulations were performed for basins of different sizes and different spatial distributions of their hydrologic parameters. It was found that, for spatially uniform hydrodynamic parameters, the effect of hydrodynamic dispersion decreases compared to that of advection as the watershed size increases, and vice versa; and that, for nonuniform hydrodynamic parameters, in conjunction with the watershed size, the spatial distribution of the parameter values over the watershed determines which process—advection or hydrodynamic dispersion—prevails.

Background

In the last decades, many researchers have devoted significant efforts to understand the effect of the watershed geomorphology on its hydrologic response. A number of models use Strahler
(1964) stream orders to describe the watershed network geomorphology. Rodriguez-Iturbe and Valdes (1979) developed a model in which the time a rainfall drop takes to reach the watershed outlet is equal to the sum of the times spent in each state (i.e., the Strahler order of the stream where the drop is located) on its way to the outlet. In their model, as well as in other models based on Strahler stream orders, one single average stream length and one single average length to the outlet are defined to describe all streams of the same order, without differentiating among them. Furthermore, spatially variable hydrodynamic coefficients and excess precipitation are not accounted for. Likewise, using a model also based on Strahler stream orders, Rinaldo et al. (1991) show that the response of a drainage area results from the combination of two distinct processes: One accounting for the flow hydrodynamics within the individual flow paths and the other for the geomorphology of the basin, which they call geomorphic dispersion. In their paper, they conclude that “models of the hydrologic response based on accurate specification of the geometry and the topology of the network, and simplified dynamics, are theoretically validated irrespective of the choice of travel time probability density functions.” Like Rodriguez-Iturbe and Valdes’ (1979) model, Rinaldo et al.’s (1991) does not support spatially varying hydrodynamic coefficients either. Botter and Rinaldo (2003) add generality to the approach of Rinaldo et al. (1991) by including a separate hillslope component in which spatial variability of the wave celerities is accounted for. Likewise, in trying to express the geomorphology of a catchment for determining its hydrological response, Snell and Sivapalan (1994) examine three different approaches to estimate network parameters: Strahler ordering, Strahler ordering with Horton order ratios, and distance–area functions derived from digital elevation models (DEMs). They conclude that the geodispersion coefficient, as defined by Rinaldo et al. (1991), derived from DEM-based distance–area functions “expresses the natural dispersion within the catchment in a more fundamental manner than the other methods.” Saco and Kumar (2002a,b) further developed this type of model by allowing the hydrodynamic parameters to vary based on the stream Strahler order. Even though their approach does not differentiate between the links of the same order (i.e., hydrodynamic parameters do not depend on local hydrologic conditions), it does advance with respect to previous models in which all links in the network had the same parameters. In their paper, it is shown that, in addition to the hydrodynamic and geomorphic dispersion mechanisms, a third mechanism is driven by the spatial variability of the hydrodynamic parameters, which they call kinematic dispersion. However, commenting on the use of Strahler stream orders to represent the watershed, Saco and Kumar (2002a) indicate that “the averaging involved in the use of a Horton-Strahler ordering scheme wipes out some of the natural variability of the topologic and geographic organization present in river networks.”

Other models use network and hillslope response functions to describe the hydrologic processes in the watershed. The catchment response is calculated as the convolution of these two responses. According to Mesa and Mifflin (1986), Naden (1992), Troch et al. (1994), and Robinson et al. (1995), the network response is the solution of the advection–dispersion equation weighted by the network width function. Naden (1992), in particular, considers an additional weighting factor that accounts for the spatial distribution of the excess rainfall; while the others support only uniform rainfall excess distributions. Likewise, for the hillslope response, each group of researchers suggests a different function, indicating that less is known about this type of flow. Moreover, in trying to relax the linearity constraint of the model, Robinson et al. (1995) define different response functions for different excess precipitation values. After applying their model to watersheds of areas 0.87, 8.7, and 87 km², they conclude that small catchments are governed by the hillslope response, while large catchments by the network response. However, because the number of stream links in the watersheds were not the same (i.e., approximately 1, 10, and 100 links for the 0.87-., 8.7-., and 87-km² watersheds, respectively), it is unclear if using networks with the same number of links would have yielded comparable results. In other words, it is not clear if small watersheds are governed by the hillslope response because of the small number of stream links in the network, or because in short flow paths hydrodynamic dispersion is more important than advection (even though hydrodynamic dispersion coefficients tend to increase with watershed size). These network and hillslope models support the use of two sets of hydrodynamic parameters: One for the network and other for the hillslope.

The Strahler order stream link classification has been criticized by Gupta and Waymire (1983) who proved that network width functions lead to better predictions of the channel response, and by Snell and Sivapalan (1994) who indicate that the “frequencies, mean lengths, and mean areas of the streams based on Strahler ordering are very dependent on the threshold area chosen for an extraction.” Likewise, Rinaldo et al. (1995) successfully determined the basin boundaries from the low-frequency component of the width function, but indicated the difficulty for accomplishing the same task from the basin response. This difficulty, according to them, is caused by the loss of information produced by the runoff processes. Both the Strahler order system and the width function, however, describe the network geomorphology by averaging all stream links of the same order or located at the same distance to the outlet without differentiating among them. This averaging, in fact, keeps the modeler from a complete accounting of the spatial variability of the hydrodynamic parameters.

While acknowledging the Strahler order stream classification and width function models, the model presented here proposes an alternative representation of the watershed. It improves with respect to previous work for modeling the watershed response because it accounts for the spatial variability of the drainage system in terms of hydrodynamic parameters and excess precipitation in a more comprehensive way. Under this approach, each point of the watershed can have different hydrodynamic parameters and excess precipitation depths. Supporting spatial variability of the hydrodynamic parameters is considered fundamental since these parameters are very sensitive to local hydrologic conditions and averaging them might overlook important features of the system.

**Watershed Response Model**

According to Maidment et al. (1996), Olivera and Maidment (1999), and Olivera et al. (2000), a watershed can be represented by a linear model in which the drainage area is subdivided into a set of nonoverlapping space-filling subareas that constitute sources of flow at the outlet, and the hydrograph at the basin outlet can be calculated as the sum of the contributions of all sources, giving

\[ Q(t) = \sum_{j} Q_j(t) \]  

(1)

where \( t \) [T] = time; \( Q(t) \) [L³/T] = hydrograph at the outlet; \( Q_j(t) \) [L³/T] = contribution of source \( j \); and the sum applies to all sources of the basin. Likewise, the flow processes in the flow path
from source \( j \) to the outlet can be represented by a time-invariant response function. Thus, \( Q_j(t) \) can be calculated as

\[
Q_j(t) = A_j R_j(t) \ast u_j(t)
\]

(2)

where \( A_j \) [L^2]=area of source \( j \); \( R_j(t) \) [L/T]=time series of excess precipitation depth at source \( j \); \( u_j(t) \) [1/T]=response function at the watershed outlet produced by an instantaneous unit input (i.e., excess precipitation depth) at source \( j \); and \( \ast \) stands for convolution. The linearity assumption, which allows for superposition of responses (Dooge 2003), is common to most instantaneous unit hydrograph models, including those discussed in the previous section. The approach presented in Eqs. (1) and (2) is particularly suitable when using raster representations of the terrain, say DEMs. In such a case, each source can be represented by a grid cell, and each flow path by a cascade of cells that connect the source to the outlet along the direction of steepest descent (Jenson and Domingue 1988). For the above reasons, in the following, the term cell is used instead of source.

The time spent by a water particle in the flow path from cell \( j \) to the outlet cell (i.e., flow-path) \( j \) can be represented by a random variable \( Y_j \) with probability density function (PDF) \( u_j(t) \); and the time spent in each cell of flow-path \( j \) can be represented by random variable \( X_k \) with PDF \( v_j(t) \). If, additionally, random variables \( X_k \) are considered independent of each other, then \( Y_j = \sum_{k=1}^{n_j} X_k \) and \( u_j(t) = v_1(t) \ast v_2(t) \ast \ldots \ast v_{n_j}(t) \) (DeGroote 1986), where \( n_j \) is the number of cells in flow-path \( j \), and the summation and convolutions apply to all cells \( k \) of flow-path \( j \). From the physical point of view, the assumption of independent random variables \( X_k \) implies that fast (or slow) water particles in one cell are not necessarily so in other cells of the flow path. In other words, the velocity of a water particle in a cell is a random variable that depends on the cell, and not a property of the particle. It also follows from the previous equations that the first and second moments of random variables \( Y_j \) and \( X_k \) are related by the following expressions: \( E(Y_j) = \sum_{k=1}^{n_j} E(X_k) \) and \( \text{Var}(Y_j) = \sum_{k=1}^{n_j} \text{Var}(X_k) \) (DeGroote 1986), where \( E \) refers to the expected value and \( \text{Var} \) to the variance.

If the flow from a cell to its immediate downstream cell is assumed to be one dimensional with no lateral inflow and the inertial terms of the St. Venant momentum equation are neglected, then it can be modeled with the diffusion equation method (Miller and Cunge 1975; Lettenmaier and Wood 1992)

\[
\frac{\partial q_k(x,t)}{\partial t} + c_k \frac{\partial q_k(x,t)}{\partial x} = -D_k \frac{\partial^2 q_k(x,t)}{\partial x^2} = 0
\]

(3)

where \( x \) [L]=distance along the segment from the center of cell \( k \) to the center of its downstream cell; \( q_k(x,t) \) [L^3/T]=flow in the segment that connects the two cells; \( c_k \) [L/T]=wave velocity in cell \( k \); and \( D_k \) [L^2/T]=hydrodynamic dispersion coefficient in cell \( k \). Assuming a system bounded by a transmitting upstream and an absorbing barrier downstream, the solution of the diffusion equation at the downstream point of the segment for an instantaneous unit input at the upstream point of the segment is (Nauman 1981)

\[
q_k(l_k,t) = \frac{1}{2 \sqrt{\pi (t/t_k) P_k}} \exp\left\{ -\frac{[1-(t/t_k)]^2}{4(t/t_k) P_k} \right\}
\]

(4)

where \( t_k = l_k/c_k \) [T]; \( P_k = c_k l_k/D_k \) [dimensionless]=Péclet number in cell \( k \); and \( l_k \) [L]=distance from the center of cell \( k \) to the center of its downstream cell. Note that, because \( q_k(l_k,t) \) is the response at the downstream point of the segment to an instantaneous unit input at its upstream point, it is also the distribution of residence times in cell \( k \); that is, \( q_k(l_k,t) \). For the distribution presented in Eq. (4), called first-passage-time distribution, \( E(X_k)=t_k \) and \( \text{Var}(X_k) = 2 t_k^2 / P_k \), which are equivalent to \( E(Y_j)=\sum_{k=1}^{n_j} (1/c_k) l_k \) and \( \text{Var}(Y_j)=\sum_{k=1}^{n_j} [2 D_k/c_k]^2 l_k \). Thus, it follows that the first and second moments of the flow-path response function \( u_j(t) \) are \( E(Y_j) = \sum_{k=1}^{n_j} (1/c_k) l_k \) and \( \text{Var}(Y_j) = \sum_{k=1}^{n_j} [2 D_k/c_k]^2 l_k \), which have to be calculated taking into consideration that the hydrodynamic parameters \( c_k \) and \( D_k \) can have different values in each cell of the flow path.

Finally, the flow-path response function \( u_j(t) \), whose exact expression results from the convolution of the \( v_j(t) \) distributions, can be approximated by a first-passage-time distribution with the same first and second moments \( E(Y_j) \) and \( \text{Var}(Y_j) \), giving

\[
u_j(t) = \frac{1}{2 \sqrt{\pi (t/T_j) P_j}} \exp\left\{ -\frac{[1-(t/T_j)]^2}{4(t/T_j) P_j} \right\}
\]

(5)

where \( T_j = E(Y_j) \) and \( P_j = \text{Var}(Y_j) / E(Y_j) \). This approximation is supported by the fact that, according to the central limit theorem (DeGroote 1986), the PDF of the sum of a number of random variables tends to normal as the number of random variables increases, and so does the distribution of Eq. (5) when the value of \( P_j \) increases as more cells are included in the flow path. Combining equations, parameters \( T_j \) and \( P_j \) can be expressed as

\[
T_j = \sum_{k=1}^{n_j} \frac{1}{c_k}
\]

(6)

\[
P_j = \left[ \sum_{k=1}^{n_j} \frac{1}{c_k} \right]^2 \sum_{k=1}^{n_j} \frac{D_k}{c_k^2} l_k
\]

(7)

From the physical point of view, \( T_j \)=expected time a water particle takes to flow from cell \( j \) to the outlet cell; and \( P_j \)=representative Péclet number for flow-path \( j \). Although the summations \( \sum_{k=1}^{n_j} (1/c_k) l_k \) and \( \sum_{k=1}^{n_j} (D_k/c_k^2) l_k \) in Eqs. (6) and (7) involve a significantly large number of hydrodynamic parameters (i.e., one wave celerity \( c_k \) and one hydrodynamic dispersion coefficient \( D_k \) per cell) and topographic data for defining the flow paths, they can be automatically calculated for all cells using geographic information systems. A function for calculating weighted flow lengths can be used to evaluate the distance from each cell to their most downstream cell in the DEM along the path of steepest descent. This distance is calculated as the sum of the lengths in each cell of the flow path. Additionally, if the lengths in each cell are multiplied by a factor (i.e., a weighting factor) \( 1/c_k \) or \( D_k/c_k^2 \), then \( \sum_{k=1}^{n_j} (1/c_k) l_k \) and \( \sum_{k=1}^{n_j} (D_k/c_k^2) l_k \) are obtained. A weight of one would produce a simple flow distance calculation. Note that for the particular case of uniformly distributed hydrodynamic parameters, Eqs. (6) and (7) become \( T_j = (1/c_l) L_j \) and \( P_j = (c/D_l) L_j \), where \( c \) [L/T] is the uniform wave celerity, \( D \) [L^2/T] is the uniform hydrodynamic dispersion coefficient, and \( L_j = \sum_{k=1}^{n_j} l_k \) is the length of flow-path \( j \).

After accounting for the contribution of all grid cells of the watershed, the mean and variance of the watershed response \( Q(t) \) produced by an instantaneous and uniformly distributed excess precipitation input are

\[
M = \bar{T}_j
\]

(8)
\[ V = \bar{T}_j^2 - \bar{T}^2_j + 2\bar{T}_j/\Pi_j \]  

where \( M \) is the variance of the watershed response; and the bar refers to average over the watershed cells. For uniformly distributed hydrodynamic parameters, Eqs. (8) and (9) become 
\[ M = L/c \text{ and } V = [\bar{T}_j^2 - \bar{T}^2_j + (2D/c)\bar{L}_j]/c^2. \]  
As mentioned above, nonuniformly distributed excess precipitation input is also supported by the model, but, since the focus of this study is on the effect of the watershed size and spatial variability of the hydrodynamic parameters, and not on the spatial distribution of the excess precipitation, Eqs. (8) and (9) have been derived for uniform excess precipitation input.

From the physical point of view, the mean \( M \) of the watershed response is the expected time a water particle takes to reach the outlet, and the variance \( V \) is a measure of the spreading of the actual times around the mean. In Eq. (8), it can be seen that \( M \) depends only on the distribution of the cell flow times to the outlet \( T_j \) (i.e., advective processes). On the contrary, in Eq. (9), the first two terms of \( V \) depend only on \( T_j \) (i.e., advective processes) and are the response variance due to advection; while the third term involves also the Péclet numbers \( \Pi_j \) (i.e., hydrodynamic dispersive processes) and are the response variance due to hydrodynamic dispersion. Note that the response variance due to advection quantifies the spreading of flow times around the mean caused by the different flow velocities and flow-path lengths and, therefore, can also be referred to as advection-induced dispersion (or geomorphologic dispersion in the particular case of uniformly distributed hydrodynamic parameters).

The comparative importance of the advective processes with respect to the hydrodynamic dispersive processes can be represented by the ratio of the first two terms of the sum of the three terms of Eq. (9), giving

\[ \Omega = \frac{\bar{T}_j^2 - \bar{T}^2_j}{\bar{T}_j - \bar{T}_j^2 + 2\bar{T}_j/\Pi_j} \]  

where \( \Omega \) is a parameter that quantifies this comparative importance. For uniformly distributed parameters, Eq. (10) becomes 
\[ \Omega = [(\bar{T}_j - \bar{T}^2_j)/(\bar{T}_j^2 - \bar{T}^2_j + 2D/c\bar{L}_j)]. \]  
Note that \( \Omega \) is the fraction of the watershed response variance that is explained by the advective processes. The value of \( \Omega \) ranges from zero for hydrodynamically disperse flow (i.e., \( \Pi_j = 0 \)) to one for advective flow (i.e., \( \Pi_j >> 1 \)). For \( \Omega \) equal to zero, responses show an instantaneous peak followed by a smooth recession curve; while for \( \Omega \) equal to one, responses show no smoothing effect of hydrodynamic dispersion and, therefore, are identical to the watershed flow time–area function (or width function in case of uniform wave celerities). Actual watershed responses, however, have values of \( \Omega \) somewhere between zero and one, for which they are smoother than the flow time–area functions, but do not have instantaneous peaks.

Furthermore, rescaled representations of actual watersheds allow one to study the effect of size on the hydrologic response, without changing the basin shape and flow patterns. Rescaling watersheds is possible because of their fractal structure, which makes fine and gross river network representations statistically indistinguishable (Rodriguez-Iturbe and Rinaldo 1997), or, as indicated by Rinaldo et al. (1995), “without a ruler, one cannot distinguish networks extracted from very large or small basins.” For rescaled watersheds, the value of \( \Omega \) of one watershed can be estimated from the value of \( \Omega \) of another homologous watershed with

![Fig. 1. Niger River basin (2,260,000 km²) and Nile River basin (3,250,000 km²) in Africa](image)

\[ \Omega_2 = \frac{1}{1 + \frac{\alpha_P}{\alpha_c} \left( \frac{1}{\Omega_1} - 1 \right)} \]  

where \( \Omega_1 \) is the dimensionless value of \( \Omega \) for watershed “1”; \( \Omega_2 \) is the dimensionless value of \( \Omega \) for watershed “2”; \( \alpha_P = L_2/L_1 \); \( \alpha_c = \text{length scale factor} \); \( \alpha_P = D_2/D_1 \); \( \alpha_c = \text{dimensionless} \); \( \alpha_c = c_2/c_1 \); \( \alpha_c = \text{ratio of the hydrodynamic dispersion coefficients in the two watersheds}; \) and \( \alpha_c = \text{ratio of the wave celerities in the two watersheds}. \) Note that, in case of nonuniform parameters, \( \alpha_P \) and \( \alpha_c \) can only be defined if the ratios are the same for any two corresponding points throughout the watershed. The ratios \( \alpha_P \) and \( \alpha_c \) can always be calculated in case of uniform hydrodynamic parameters.

### Application and Discussion

The Niger River and Nile River basins in Africa (Fig. 1) were used for the simulations. These two basins were selected because of their very different geomorphologies: The Niger basin is comprised of three major branches conveying flow from different areas, while the Nile basin consists predominantly of one major river channel. Their catchment areas are 2,260,000 km² and 3,250,000 km², respectively (Revenga et al. 1998). In this study, however, the actual hydrologic characteristics of the basins were not used, with the exception of their shapes and flow patterns. Wave celerities and hydrodynamic dispersion coefficients used in the simulations fell within the ranges documented by Fisher et al. (1979) and Deng et al. (2001). The geographic data were derived from the HYDRO1K DEM of Africa (Gesch et al. 1999) developed by the U.S. Geological Survey Earth Resources Observation Systems Data Center, which has a horizontal resolution of 1 km.

For each simulation, a two-column table with the values of \( \sum_{i=1}^{n_i}(1/c_i)l_i \) and \( \sum_{i=1}^{n_i}(D_i/c_i^2)l_i \) of each grid cell was prepared by applying the weighted flow length function to the HYDRO1K DEM as discussed above. This table was then used as input to an application that calculates \( T_j \) and \( \Pi_j \) using Eqs. (6) and (7), the cell response functions \( u(t) \) using Eq. (5), the cell contributions \( Q_j(t) \) using Eq. (2), and the watershed response \( Q(t) \) using Eq. (1).
Note that, when applying Eq. (2), all cells have the same area and excess precipitation input.

**Uniform Hydrodynamic Parameters**

The assumption of uniform parameters is more a mathematical exercise to help one better understand the effect of each variable on the watershed response than a real description of the landscape. Given that the hydrodynamic parameters are assumed uniform, the advection-induced dispersion in the watershed response is equivalent to the geomorphologic dispersion found in the literature. To assess the effect on the watershed response of changes in the drainage area size and hydrodynamic parameters, without interference of other hydrologic effects, all other parameters were kept unchanged.

For a wave celerity of 1 m/s and hydrodynamic dispersion coefficient of 150 m²/s, Fig. 2(a) shows the responses of the Niger River basin affected by length scale factors of 1, 0.1, 0.01, and 0.001, so that the drainage areas were 2,260,000, 22,600, 226, and 2.26 km². Likewise, Fig. 2(b) shows the responses of the Nile River basin affected by scale factors of 0.84, 0.084, 0.0084, and 0.000084, so that the drainage areas were the same as in the Niger case. In both cases, it was noticed that the larger catchments were dominated by advection with values of Ω close to 1; and that their responses became smoother as the area decreased. The smaller basins, however, exhibit the effect of both advection-induced and hydrodynamic dispersion with values of Ω of 0.57 (Niger) and 0.66 (Nile), and their responses were smooth with shorter peak times and longer recession curves. Note that the four responses in each figure are noticeably different among them, although the four watersheds have the same shape, flow-path patterns, number of stream network links, and hydrodynamic parameters, but different sizes.

Fig. 3(a) presents the responses of the Niger River basin for a scale factor of 1 (i.e., drainage area of 2,260,000 km²), wave celerity of 1 m/s, and hydrodynamic dispersion coefficients of 15, 150, and 1,500 m²/s; and Fig. 3(b) for a scale factor of 0.001 (i.e., drainage area of 2.26 km²) and same hydrodynamic parameters. In the former case, it can be noticed that not even considerably large values of the hydrodynamic dispersion coefficient kept the advective processes from dominating. It is clear, from the shape of the responses, that their overall shape is similar, but also that for 1,500 m²/s the response is much smoother than for the lower dispersion coefficients. In the latter case, the shapes of the responses were significantly different among them because of the
different mechanism that dominated. For 15 m$^2$/s, although smooth, the response was dominated by advection; while for 1,500 m$^2$/s, it was dominated by hydrodynamic dispersion. According to Fig. 3, small watershed responses are more sensitive to changes in the hydrodynamic dispersion coefficients than large watersheds.

Finally, Fig. 4(a) shows the value of $\Omega$ for different watershed sizes, wave celerity of 1 m/s, and hydrodynamic dispersion coefficients of 15, 150, and 1,500 m$^2$/s, for the Niger and Nile basins; and Fig. 4(b) for wave celerities of 0.33, 1.00, and 3.00 m/s, and hydrodynamic dispersion coefficient of 150 m$^2$/s. For uniformly distributed hydrodynamic parameters, it can be proved that $\overline{T_j^2} - \overline{T_j^2}$ (i.e., the response variance caused by advection) tends to increase with $\overline{T_j}$, while $2\overline{T_j}/\Pi_j$ (i.e., the response variance caused by hydrodynamic dispersion) with $\overline{T_j}$. Therefore, as the watershed size and the length of the flow paths increase, advection-induced dispersion (or geomorphologic dispersion given that uniform hydrodynamic parameters are being considered) tends to overwhelm hydrodynamic dispersion, and vice versa.

**Nonuniform Hydrodynamic Parameters**

In general, higher hydrodynamic dispersion coefficients are expected in larger streams, with deeper and wider cross sections, than in small streams or overland flow. The same, however, does not always hold true for flow velocities. Although as flows increase so do flow velocities, it is also true that as rivers approach the ocean and the channel slope decreases, flows slow down. Overall, hydrodynamic parameters are very sensitive to local conditions, and it is difficult to propose general rules to estimate them for all points of the landscape. Nonetheless, for the simulations included below, equations for estimating realistic values of the hydrodynamic parameters were used. These equations, however, should not be implemented in real-world cases without further case-specific verification.

**Hydrodynamic Parameters based on Regression Equations**

Regression equations based on data documented by Deng et al. (2001) were used for estimating wave celerities and hydrodynamic dispersion coefficients. The equations were $c_j = 0.227Q_j^{0.2154}$ ($r^2=0.54$) and $D_j = 11.505Q_{j}^{0.4951}$ ($r^2=0.69$), where $c_j$ (m/s), $D_j$ (m$^2$/s), and $Q_j$ (m$^3$/s) are the wave celerity, hydrodynamic dispersion coefficient, and flow in cell $j$, respectively. These equations considered 70 sets of values in which each set included the channel depth and width, flow velocity, and dispersion coefficient. The flow was calculated as the product of the channel width and depth, flow velocity, and dispersion coefficient. The flow was calculated as the product of the channel width and depth, and the flow velocity. The relatively low values of $r^2$ indicate the partial dependence of the parameters on variables other than flow. Additionally, if the assumption of flow proportional to the drainage area is made, the regression equations can be written in terms of drainage area rather than flow. For the simulations presented here, it was assumed that $Q = 0.00828 \times A_j$, where $A_j$ is the drainage area at the point in km$^2$. This assumption kept the values of the dispersion coefficients below 1,500 m$^2$/s, which is consistent with Fisher et al.’s (1979) and Deng et al.’s (2001) data.

A first set of simulations was performed assuming constant wave celerities of 1 m/s and variable dispersion coefficients for both the Niger and Nile River basins and different length scale factors (i.e., 1.0, 0.3, 0.1, 0.03, 0.01, 0.003, and 0.001). Fig. 5 (two lower curves) shows the resulting $\Omega$ values, and Fig. 6, the responses. Similarly, a second set of simulations assumed variable
wave celerities and dispersion coefficients. Fig. 5 (two upper curves) shows the $\Omega$ values, and Fig. 7, the responses.

In Fig. 5, it was interesting to notice the effect of keeping uniform or varying the wave celerities. In the case of uniform wave celerities, the tendency of advection-induced dispersion to increase with the basin size remained valid and comparable to that previously observed for uniform hydrodynamic parameters. That tendency, however, was significantly less clear in the case of varying wave celerities. The explanation of this difference is that, for nonuniform wave celerities, in our simulations, the flow was noticeably slower in the smaller basins, which increased the response variance. However, although this increase was caused by advection-induced and hydrodynamic dispersion effects, it was dominated by the wider distribution of flow times as opposed to the more concentrated one found with the uniform wave celerity of 1 m/s; that is, it was dominated by advection. In Figs. 6 and 7, it can be seen that responses dominated by advection (i.e., high values of $\Omega$) have a common overall shape, although some of them are smoother than others. Likewise, responses dominated by hydrodynamic dispersion tend to be smooth with a shorter peak time and long recession curve.

Different results can be expected for parameter distributions other than those based on the regression equations presented above. In fact, these regression equations do not account, for example, for the slowing down that takes place in a river as it approaches the ocean or for areas of significant floodplain storage (i.e., slow celerities and high hydrodynamic dispersion). The effect of areas of particular hydrologic conditions is discussed below.

**Effect of Local Hydrologic Conditions**

Simulations assuming variable wave celerities and dispersion coefficients, according to the regression equations presented above, plus the effect of localized areas of particular hydrologic conditions, were also performed. For the Niger basin, these areas were the inner delta located upstream of Tombouctou in Mali, which approximately drains 400,000 km$^2$ and has a length of 350 km along the main river channel, and the river delta at the Atlantic Ocean, which drains the entire basin and has channel lengths of approximately 200 km. For the Nile basin, these areas were the Sudd Marshes located in southern Sudan, which approximately drain 1,600,000 km$^2$ and have a length of 300 km along the main
river channel, and the river delta at the Mediterranean Sea, which drains the entire basin and has channel lengths of approximately 200 km. In these areas, wave celerities were taken as 0.10 m/s and dispersion coefficients as 1,500 m²/s [i.e., the greatest value documented by Fisher et al. (1979)]. Note that, because of the small size of these areas (i.e., around 1% of the watershed area), their effect is very much that of concentrated points in which water is stored and delayed. All simulations assumed a length scale factor of 1, which implies the actual size of the watershed. Fig. 8 shows the response functions and the Ω values for the following cases: (1) Basin with river delta; (2) basin with inner delta (Niger) or Sudd Marshes (Nile); and (3) basin with river delta and inner delta or Sudd Marshes. The term basin refers to the drainage area with hydrodynamic parameters based on the regression equations, and its response was plotted as a base case for comparison.

For the Niger base case, $\overline{T_j} - T_j = 64$ days² (i.e., the response variance caused by advection-induced dispersion), $2\overline{T_j} / \Pi_j$ was negligible (i.e., the response variance caused by hydrodynamic dispersion) and $\Omega = 0.997612$.

Based on Fig. 8, the effect of the river delta was apparent. With respect to the base case, the residence time in the basin increased by an amount equal to the residence time in the delta, causing almost the entire flow time distribution to shift to the right without visibly changing the value of $\overline{T_j} - T_j$. Thus, the response variance caused by advection remained virtually the same. On the contrary, the slow wave celerities and high dispersion coefficients in the delta significantly increased the value of $2\overline{T_j} / \Pi_j$, increasing the response variance caused by hydrodynamic dispersion to 81 days². Consequently, the value of $\Omega$ decreased to 0.442 887 indicating that hydrodynamic dispersion played a more important role in this case.

The effect of the inner delta, however, was strikingly different from that of the river delta. In this case, because only part of the basin was captured, the domain of the flow time distribution significantly increased and so did $\overline{T_j} - T_j$. Consequently, the response variance caused by advection increase to 410 days². Likewise, the slow celerities and high dispersion coefficients also increased $2\overline{T_j} / \Pi_j$, but because of the smaller area captured compared to the river delta case, the response variance caused by hydrodynamic dispersion increased only to 26 days². Thus, the value of $\Omega$ decreased with respect to the base case to 0.940 780, which shows that the effect of the inner delta is qualitatively different from that of the river delta.

When considering the presence of the river delta and inner delta in the basin, the response variance took into account the delay in both combined. The response variance caused by advection was 410 days², and that caused by hydrodynamic dispersion 106 days². The value of $\Omega$ was 0.794 373, which is higher than that observed for the basin and river delta (i.e., $\Omega = 0.442 887$), implying that the inner delta increased the role of the advection-induced dispersion more than it did the role of hydrodynamic dispersion.

For the Nile basin, $\overline{T_j} - T_j = 103$ days², $2\overline{T_j} / \Pi_j$ was negligible and $\Omega = 0.998 026$ for the base case; for the basin and river delta, $\overline{T_j} - T_j = 103$ days², $2\overline{T_j} / \Pi_j = 81$ days² and $\Omega = 0.561 524$; for the basin and Sudd Marshes, $\overline{T_j} - T_j = 443$ days², $2\overline{T_j} / \Pi_j = 26$ days² and $\Omega = 0.943 703$; and for the basin, river delta and Sudd Marshes, $\overline{T_j} - T_j = 443$ days², $2\overline{T_j} / \Pi_j = 107$ days² and $\Omega = 0.805 645$. Overall, these numbers observe the same patterns as those of the Niger basin.

Note that, for nonuniform hydrodynamic parameters, not only does the value of $\Omega$ depend on the watershed size, but also on the spatial distribution of the hydrodynamic parameters. In our simulations, it was observed that a uniform and a regression-equation-based spatial distribution of wave celerities produced dramatically different responses for a wide range of watershed sizes, despite the identical spatial distributions of the dispersion coefficients. In fact, the tendency of hydrodynamic dispersion to become predominant as the watershed size decreases was not as strong when using uniform wave celerities and regression-equation-based dispersion coefficients as that observed for uniform parameters. It was also observed that local hydrologic conditions that affect the values of wave celerities and dispersion coefficients can severely affect the response of the entire watershed. Furthermore, how these local hydrologic conditions affect the response depends on where in the basin they are found. In fact, it was observed that the effect of areas of slow flow and high dispersion coefficients was significantly different depending on the fraction of the basin they captured. River deltas, for example, which capture the entire basin, tend to increase the variance caused by hydrodynamic dispersion, but only negligibly that caused by advection. As the frac-

Fig. 8. Watershed responses for a length scale factor of 1 and different cases of areas of unique hydrologic conditions: (a) Niger basin: $\Omega = 0.997612$ for the basin, $\Omega = 0.442887$ for the basin and delta, $\Omega = 0.940780$ for the basin and inner delta, and $\Omega = 0.794373$ for the basin, delta and inner delta; and (b) Nile basin: $\Omega = 0.998289$ for the basin, $\Omega = 0.561524$ for the basin and delta, $\Omega = 0.943703$ for the basin and Sudd Marshes, and $\Omega = 0.805645$ for the basin, delta, and Sudd Marshes.
tion of the basin captured decreased, its effect on the variance caused by hydrodynamic dispersion decreased because it affected less area, but its effect on the variance caused by advection increased as long as it widened the flow time distribution (i.e., as long as it increased $\overline{t^2} - \overline{t}^2$). In our simulations, the Niger inner delta and Nile Sudd Marshes increased the response variance caused by advection more than that caused by hydrodynamic dispersion, which is opposite to what was observed in the river delta case. Additionally, based on our simulations, it is clear that large watersheds can also have a strong component of hydrodynamic dispersion and that it would be inaccurate to assume that large watersheds, just because of their size, are dominated by advection. Thus, a priori estimations of the relative importance of advection with respect to hydrodynamic dispersion in nonuniform terrain are not possible and a detailed analysis of the watershed response is necessary.

Conclusions

A spatially distributed hydrologic model for estimating the watershed response was presented. The model subdivides the watershed into nonoverlapping space-filling areas, and estimates a response function for each of them based on the flow time (i.e., first moment) and spreading around the centroid due to hydrodynamic dispersion (i.e., second moment) in the flow path. The contribution of an area to the flow at the outlet is calculated as the convolution of the excess precipitation input and its response function. The watershed hydrograph, in turn, is the sum of the contributions of the areas. The model supports the use of nonuniform hydrodynamic parameters, which allow one to account for specific local hydrologic conditions.

The watershed model was used to assess the effect of advective and hydrodynamic dispersive processes on the watershed hydrologic response. A parameter $\Omega$ was defined to quantify the comparative importance of advective with respect to hydrodynamic dispersive processes. This parameter represents the fraction of the watershed response variance that is explained by advection, and equals one for completely advective flow and zero for completely dispersive flow. Basin responses that mimic the watershed flow time–area function correspond to values of $\Omega$ close to 1, its downstream cell

For uniformly distributed hydrodynamic parameters, it was found that, as the watershed size increases, advection becomes more important than hydrodynamic dispersion; and that, as the watershed size decreases, responses become dominated by hydrodynamic dispersion. In the simulations presented above, noticeably different response shapes were found for watersheds that had the same shape, flow-path patterns, number of stream network links, and hydrodynamic parameters, but different sizes.

For nonuniformly distributed hydrodynamic parameters, it was found that, in addition to the watershed size, the spatial variability of the hydrodynamic parameters plays a fundamental role in determining if advective or dispersive processes dominate the watershed response. The spatial variability of these parameters, however, is difficult to estimate. Even though wave celerities and dispersion coefficients tend to increase in the downstream direction, local hydrologic conditions can define areas of unique hydrodynamic parameters. In the simulations, it was observed that the effect of slow-flow and high-dispersion areas affected the watershed response differently depending on their location in the watershed, and on the fraction of the drainage area they captured. Additionally, based on our simulations, it is clear that large watersheds can also have a strong component of hydrodynamic dispersion and that it would be inaccurate to assume that large watersheds, just because of their size, are dominated by advection. Thus, a priori estimations of the relative importance of advection with respect to hydrodynamic dispersion in nonuniform terrain are not possible, and a detailed analysis of the watershed response is necessary.

Notation

The following symbols are used in this paper:

- $A_j =$ area of source or cell $j \, [L^2]$;
- $A_j^f =$ drainage area at cell $j \, [L^2]$;
- $c =$ uniform wave celerity [L/T];
- $c_k =$ wave celerity in cell $k \, [L/T]$;
- $D =$ uniform hydrodynamic dispersion coefficient $[L^2/T]$;
- $D_k =$ hydrodynamic dispersion coefficient in cell $k \, [L^2/T]$;
- $j =$ source or cell index;
- $k =$ cell of flow-path index;
- $L_j =$ length of the flow path from source or cell $j$ to the outlet [L];
- $l_k =$ distance from the center of cell $k$ to the center of its downstream cell [L];
- $M =$ mean of the watershed response [T];
- $n_j =$ number of cells in the flow path from source or cell $j$ to the outlet;
- $P_k =$ Péclet number in cell $k$;
- $Q(t)$ =$ hydrograph at the watershed outlet $[L/T^3]$;
- $Q(t)$ =$ contribution of source or cell $j$ to the hydrograph $[L/T^3]$;
- $q_k(x,t) =$ flow in the segment that connects cell $k$ with its downstream cell $[L^3/T]$;
- $R_j(t)$ =$ time series of runoff depth at source or cell $j \, [L/T]$;
- $V =$ variance of the watershed response [T];
- $t =$ time [T];
- $T_j =$ expected time spent by a water particle in the flow path from source or cell $j$ to the outlet [T];
- $t_k =$ expected time spent by a water particle in cell $k$ [T];
- $u_j(t)$ =$ response at the watershed outlet to an instantaneous unit input at source or cell $j$. Also, probability density function of random variable $Y_j(1/T)$;
- $v_k(t)$ =$ probability density function of random variable $X_k(1/T)$;
- $X_k =$ random variable that represents the time spent by a water particle in cell $k$ [T];
- $x =$ distance along the segment that connects cell $k$ with its downstream cell [L];
- $Y_j =$ random variable that represents the time spent by a water particle in the flow path from source or cell $j$ to the outlet [T];
- $\alpha_D =$ ratio of the hydrodynamic dispersion coefficients of two homologous watersheds;
- $\alpha_L =$ ratio of the lengths of two homologous watersheds;
\( \alpha_c \) = ratio of the wave celerities of two homologous watersheds;

\( \Pi_i \) = representative Péclet number in the flow path from source or cell j to the outlet; and

\( \Omega \) = parameter that quantifies the comparative importance of advection and hydrodynamic dispersion.

References


