CVEN 311-501 “Fluid Dynamics”

**H O M E W O R K  6**

_Due Friday, 10/19/2012 at 8:00 AM_  
(No late papers accepted without documented university excuse)

**PART I** – Optional textbook problems. Do not turn these problems in with your assignment.

*Fluid Kinematics – Velocity and Acceleration Fields*
1. Textbook problem 4.5, pg. 190 (6th ed.: 4.6, pg. 179)

*Continuity – Steady Flow with Uniform and Non-uniform Velocity Profiles*
PART II – Required problems. You must turn these problems in with your assignment. All multiple choice problems should be answered with a complete solution; simply stating a letter answer will receive no credit. All problems will be graded on effort alone.

Fluid Kinematics – Velocity and Acceleration Fields

1. Pump stations are sometimes designed with submerged vertical inlets where water is sucked upward from a wet well through a bell-shaped inlet into a riser pipe. A key concern in design of these facilities is avoidance of excessive rotation in the flow traveling up the riser pipe. Rotating flows will lessen the efficiency of the rotating pump impellor and can lead to formation of vortices that can suck air into the line that can significantly damage the pump. To avoid problems, pump station designs are often tested with physical models. A $30 million pump station expansion recently built in Las Vegas was model tested at TAMU in 2008.

The engineering standard governing design and testing of pump intakes is ANSI/HI 9.8-1998 (often shortened to “HI 9.8”). This standard specifies that a model riser should include a “swirl meter” as sketched below; also shown is a photo of a swirl meter installed in a transparent riser in the 2008 TAMU model.

HI 9.8 requires that model tests quantify swirl by computing a “swirl angle” that is the angle between a typical fluid particle’s velocity vector and the vertical axis of the riser. The swirl angle $\alpha$ is computed as:

$$\alpha = \tan^{-1}\left( \frac{\pi dn}{v_z} \right)$$

where $d$ is the diameter of the riser pipe at the swirl meter, $n$ is the rotational speed of the swirl meter in revolutions per second, and $v_z$ is the average velocity of the flow along the axis of the riser. Thus, the swirl angle $\alpha$ is found from a horizontal velocity component (measured by rotation of the meter) and a vertical component. HI 9.8 specifies that swirl angles must be less than 5 degrees.
Recent advances in experimental methods can allow precise determination of the velocity field through high-speed photography of dyes and particles in the flow. These techniques could replace the swirl meters currently used, which would be good since there is little uniformity in how they are fabricated. A recent model test used such a technique and determined the steady flow velocity field in a 0.40 ft diameter riser in cylindrical coordinates $(r, \theta, z)$:

$$v_r = 0 \text{ ft/sec}$$

$$v_\theta = \frac{0.031}{r^{1/4}} \text{ ft/sec}$$

$$v_z = 7.6 - 47r^2 \text{ ft/sec}$$

In order to calculate the swirl angle $\alpha$, you will need to determine the average rotational speed in the horizontal plane $\overline{v_\theta}$ and the average axial speed across the riser cross-section $\overline{v_z}$. The average value theorem is a convenient way to do this. Respectively:

$$\overline{v_\theta} = \frac{1}{r_2 - r_1} \int_{r_1}^{r_2} v_\theta(r) \, dr$$

$$\overline{v_z} = \left(\frac{1}{A}\right) \int_A v_z \, dA = \left(\frac{1}{\pi r^2}\right) \int_{\theta=0}^{\theta=2\pi} \int_{r_1}^{r_2} v_z(r) r \, dr \, d\theta$$
(a) Compute the swirl angle for the flow described above in units of degrees, and determine whether the model currently meets HI 9.8 specifications.
(b) Comment on the differences likely to occur between the standard method of computing the swirl angle with a physical meter and the numerical method you have just used.

Continuity – Steady Flow with Uniform and Non-uniform Velocity Profiles

2. {From Fall 2010 Final Exam} A desalination filter has an inflow of seawater ($\gamma = 64.0 \text{ lb/ft}^3$) at a volumetric flowrate of 5000 gal/day. Two outflows leave the filter: pure water ($\gamma = 62.4 \text{ lb/ft}^3$) at 3500 gal/day and waste brine. Assuming that the volume of dissolved salt is negligible, what is the specific weight $\gamma$ of the waste brine?

(A) 88.8 lb/ft$^3$
(B) 63.2 lb/ft$^3$
(C) 64.7 lb/ft$^3$
(D) 67.7 lb/ft$^3$

3. {From Fall 2010 Final Exam} A water storage tank is a cylinder 75 ft in diameter and 32 ft high. Inflow to the tank is constant at 1200 gal/min, and the tank is empty at 12:00 Midnight. If outflow from the tank is 275 gal/min from 12:00 Midnight to 4:15 AM and zero after 4:15 AM, at what time will the tank be full of water?

(A) 11:25 AM
(B) 3:40 PM
(C) 2:41 PM
(D) 7:15 AM